Review 23 L'hôpital's Rule

indeterminate forms

$\frac{0}{0}, \frac{\infty}{\infty}$

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

other indet. forms

between $\infty - \infty$ 0^0

$0 \cdot \infty$ ∞^0

1^∞ 0^∞

transform into fraction
in order to use l'hopital's rule

not Indet.

$\frac{k}{\infty}$ $\frac{0}{\infty} \to 0$

$\frac{\infty}{k}$ $\frac{\infty}{0} \to \infty$

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Ex 1

$$\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x}$$

$$\lim_{x\to 0} \frac{\frac{1}{2}(1+x)^{-\frac{1}{2}}}{1}$$

$$\frac{1}{2}$$

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Ex 2

$$\lim_{x\to\infty} \frac{\ln x}{2\sqrt{x}}$$

$$\lim_{x\to\infty} \frac{\frac{1}{x}}{x^{-\frac{1}{2}}}$$

$$\lim_{x\to\infty} \frac{x^{\frac{1}{2}}}{x}$$

$$\lim_{x\to\infty} \frac{1}{x^{\frac{1}{2}}} = 0$$

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Ex 3

$$\lim_{x\to 0^+} \frac{\ln x}{\tan x}$$

$$\frac{-\infty}{0}$$

$$-\infty$$

no l'hopital

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Ex 4 $\lim_{x\to\infty}\left(\frac{x}{1+x}\right)^x$

remember: $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$

$$\lim_{x\to\infty}\left(\frac{x+1}{x}\right)^x = e$$

$$\lim_{x\to\infty}\left(\frac{x}{x+1}\right)^x = \frac{1}{e}$$

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$$\lim_{x\to\infty}\left(\frac{x}{1+x}\right)^x = y$$

$$\lim_{x\to\infty}\ln\left(\frac{x}{1+x}\right)^x = \ln y$$

$$\lim_{x\to\infty}\frac{\ln\left(\frac{x}{1+x}\right)}{\frac{1}{x}} = \ln y$$

$$\lim_{x\to\infty}\frac{\left(\frac{1+x}{x}\right)\cdot\frac{(1+x)\cdot 1 - x\cdot 1}{(1+x)^2}}{-\frac{1}{x^2}}$$

$$\lim_{x\to\infty}\frac{\cancel{1+x}}{\cancel{x}}\frac{(1)(-x^{\cancel{2}})}{(1+x)^{\cancel{2}}} = \lim_{x\to\infty}\frac{-x}{1+x}$$

$$-1 = \ln y$$

$$y = \frac{1}{e}$$

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